

FINAL PROJECT REPORT
of
MAJOR RESEARCH PROJECT of
UNIVERSITY GRANTS COMMISSION, NEW DELHI
[F.No. 42-02/2013(SR), 1- 4 -2013
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for
MATHEMATICS SUBJECT ENTITLED
“A study of intuitionistic fuzzy G-modules”
SUBMITTED BY:
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CERTIFICATE

I, Dr. Poonam Kumar Sharma, M.Sc., Ph.D., declare that the work presented in this report is original and carried out by me independently during the complete tenure of major research project. However some part of the work is done with the assistance of the project fellow appointed for the said purpose by U.G.C., New Delhi.

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SUMMARY OF FINAL REPORT

of

Major Research Project of UGC, New Delhi

[F.NO. 42-2/2013(SR), Dated: 14-03-2014]

In **Mathematics** entitled

“A study of intuitionistic fuzzy G-modules”

The whole work initiated and completed may be divided into four sections.

Section 1. Review work based on proposed problems

Section 2. Study of the abstract properties of the problem with respect to their intrinsic nature

Section 3. Analysis of the major problems of the project based upon the previous work done so far.

Section 4. Some supplementary part of the related problem pertaining to the project

Section 1

At the initial stages of the project, intensive literature survey based on G-modules, fuzzy modules, intuitionistic fuzzy modules, fuzzy G-modules, reducibility and complete reducibility of fuzzy modules and fuzzy G-modules, projectivity and injectivity of modules (fuzzy modules, fuzzy G-modules) have been made.

Based on these, the notion of intuitionistic fuzzy G-modules has been introduced. Many concrete examples have been constructed. Several operations like union, intersection, sum, product of two or more intuitionistic fuzzy G-modules have been defined and studied. A representation of intuitionistic fuzzy group and that of intuitionistic fuzzy G-modules has been analysed.

Three papers have been prepared over these topics.

The details are mentioned as under:

Paper 1. “Intuitionistic fuzzy G-modules” published in the Journal , **Notes on Intuitionistic Fuzzy Sets, Vol. 21, Number 1, 2015, pp. 6-23.**

In this paper, the notion of intuitionistic fuzzy G-modules M over a field K is defined. Some operations like intersection, sum and product, Cartesian product of two intuitionistic fuzzy G-modules are defined. The quotient intuitionistic fuzzy G-modules is also defined and discussed. The notion of (weak) intuitionistic fuzzy G-module homomorphism (G-isomorphism) is also introduced and analyzed. Some concrete examples in each case have been framed.

Paper 2. “Intuitionistic fuzzy representation of intuitionistic fuzzy groups”, published in the Journal, **Asian Journal of Fuzzy and Applied Mathematics, Vol. 3 , No. 3 , 2015, pp. 81-94.**

In this paper, we study the intuitionistic fuzzy representation of intuitionistic fuzzy groups. A fundamental theorem of intuitionistic fuzzy representation of quotient groups has been derived. It is proved that every intuitionistic fuzzy representation gives rise to an intuitionistic fuzzy representation on the factor groups. An intuitionistic fuzzy version of the famous Cayle’s theorem has been derived.

Paper 3. “ Intuitionistic fuzzy representation of intuitionistic fuzzy G-modules”, published in the Journal, **Annals of Fuzzy Mathematics and Informatics, Vol. 11, No. 4, 2016, pp. 557–569.**

In this paper, we study intuitionistic fuzzy representation of an intuitionistic fuzzy G-module A_N of a G-module M/N onto an intuitionistic fuzzy G-module B of a general linear space $GL(V)$. This transformation is done

on the basis of G -module representation theory. A fundamental theorem of intuitionistic fuzzy representation of G -module has been proved.

Section 2

In the second stage, we used the results obtained in stage one for studying some other properties like reducibility and complete reducibility of intuitionistic fuzzy G -modules. An important property of modules, i.e., semi-simplicity is also studied for intuitionistic fuzzy G -modules.

Two papers have been prepared over these topics.

Paper 4. “Reducibility and complete reducibility of intuitionistic fuzzy G -modules”, published in the Journal, **Annals of Fuzzy Mathematics and Informatics, Vol. 11, No. 6, 2016, pp. 885–898.**

In this paper, we analyze the reducibility and complete reducibility property of the intuitionistic fuzzy G -modules of a G -module. A method of constructing an intuitionistic fuzzy G -module M in terms of double pinned flag is also developed. By using this method, we establish that any finite dimensional G -module M of dimension at-least two, there exists infinite many completely reducible intuitionistic fuzzy completely G -modules. Further, it is noticed that the union of intuitionistic fuzzy completely reducible G -modules is an intuitionistic fuzzy completely G -module but intersection of intuitionistic fuzzy completely reducible G -modules is not an intuitionistic fuzzy completely G -module.

Paper 5. “Semi-simple intuitionistic fuzzy G -modules”, published in the Journal, **International Journal of Pure and Applied Researches, Vol. 1, No. 2, 2016, pp. 101–108.**

In this paper, the concept of semi-simplicity of modules is extended to semi-simple intuitionistic fuzzy G -modules. The existence of a semi-simple

intuitionistic fuzzy G-module of every finite dimensional G-module is proved. A relationship between semi-simplicity of intuitionistic fuzzy G-module with other properties of intuitionistic fuzzy G-modules is also obtained.

Section 3

In the third stage, we used the results obtained in first two stages for studying some other properties like injectivity and projectivity of intuitionistic fuzzy G-modules. An important property of modules, i.e., Three Isomorphism Theorems are also studied for intuitionistic fuzzy G-modules.

Three papers have been prepared over these topics.

Paper 6. “Projectivity of intuitionistic fuzzy G-modules”, published in the Journal, **Advances in Fuzzy Sets and System, Vol. 21, No. 3, 2016, 239-264.**

In this paper, we introduce the notion of projectivity of intuitionistic fuzzy G-modules. A condition is established on finite dimensional G-modules for which an intuitionistic fuzzy G-module is projective with respect to another intuitionistic fuzzy G-module. Projectivity with respect to the quotient intuitionistic fuzzy G-module of another is also discussed. Some properties of projectivity of intuitionistic fuzzy G-modules with respect to the direct sum of intuitionistic fuzzy G-modules are also studied. A relationship between the projectivity and quasi projectivity of intuitionistic fuzzy G-modules is also obtained.

Paper 7. “Injectivity of intuitionistic fuzzy G-modules”, published in the Journal, **Annals of Fuzzy Mathematics and Informatics, Vol. 12, No. 6, 2016, 805-823.**

In this paper, we introduce the notion of injectivity and quasi injectivity of an intuitionistic fuzzy G-module and have constructed some structure revealing examples. We have also analysed the relative injectivity

(quasi injectivity) of an intuitionistic fuzzy G -module with regards to another intuitionistic fuzzy G -module.

Paper 8. “Isomorphism theorems for intuitionistic fuzzy submodules of G -modules” published in the Journal , **Notes on Intuitionistic Fuzzy Sets, Vol. 22, Number 4, 2016, pp. 80-88.**

In this paper, by using the notion of intuitionistic fuzzy submodules of G -modules and the G -homomorphism as introduced in paper one, we establish three isomorphism theorems for intuitionistic fuzzy G -submodules of G -modules.

Section 4

In the fourth stage, we have made an Appendix which includes the supplementary part on the different structures involved in the project.

Four papers have been prepared for the supplementary part.

Paper 9. “ t -intuitionistic fuzzy G -modules” accepted for presentation and publication in the proceeding of the **International Conference on Differential Geometry, Algebra and Analysis** to be held at Jamia Millia Islamia University, New Delhi from 15-17th November, 2016.

In this paper, by using the notion of “ t -intuitionistic fuzzy sets w.r.t. an intuitionistic fuzzy sets” which the author introduced earlier in his work, has introduced the concept of t -intuitionistic fuzzy G -modules and studied various properties.

Paper 10. “On intuitionistic fuzzy G -modules of $GF(p^n)$ ”, published in the Journal , **Asian Journal of Fuzzy and Applied Mathematics, 4(4)(2016) 47-53.**

In this paper, we construct an intuitionistic fuzzy G -module on Galois field $GF(p^n)$ and show that infinite many such intuitionistic fuzzy G -modules

can be constructed. Noetherian structure of the intuitionistic fuzzy G-module of $GF(p^n)$ is also studied.

Paper 11. “ Direct sum of intuitionistic fuzzy submodules of G-module ,” published in the Journal , **Asian Journal of Fuzzy and Applied Mathematics, 4(4)(2016) 36-46.**

In this paper, we introduce the notion of direct sum of intuitionistic fuzzy submodules of G-modules and discuss some related properties. Some results concerning decomposability and indecomposability of intuitionistic fuzzy G-modules are also discussed.

Paper 12. “ Translates of intuitionistic fuzzy G-modules” Proceeding of the **National Conference on Recent trends and Technologies in data Science and Artificial Intelligence held at Baderwah Campus, University of Jammu from August 26-27, 2016 , page 1-6, ISBN: 978-93-5112-284-4.**

In this paper, by using the notion of “Translates of intuitionistic fuzzy sets w.r.t. an intuitionistic fuzzy sets” which the author introduced earlier in his work, has introduced the concept of Translates of intuitionistic fuzzy G-modules and studied various properties.



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Dated: Nov., 10, 2016

Major Research Project Report

The major research project titled “A study of intuitionistic fuzzy G-modules” submitted by Dr. Poonam Kumar Sharma makes a significant contribution to the intuitionistic fuzzification of the theory of G-modules. The project, mainly focuses on the study of two major classes of G-modules namely projective G-modules and injective G-modules and their intuitionistic fuzzification which lead to the study of many important properties like reducibility, complete reducibility, semi-simplicity of G-modules.

The project contains a substantial amount of work on the three fundamental theorems of isomorphisms for the intuitionistic fuzzy G-modules. Many new and useful results on the decomposition of intuitionistic fuzzy G-modules have been derived. The thesis, also, contains a supplement part of the work showing some applications to the structure of intuitionistic fuzzy G-modules.

There has been a considerable amount of study going on in this direction in recent years. The work presented by Dr. Poonam Kumar Sharma proceeds in making a good contribution to the advancement of knowledge in this field. The validity of the results have been proved in the form that most of the results are published in reputed international journals. To sum up, the work presented in the project is a significant contribution in the study of intuitionistic fuzzification of G-module.

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From: Dr. Helen K. Saikia
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07/11/2016

Report

The report on the Major research project entitled “A STUDY OF INTUITIONISTIC FUZZY G-MODULES” submitted by Dr. Poonam Kumar Sharma, contains substantial amount of work in the field of intuitionistic fuzzy G-modules. It is a good contribution to the subject and is well written. Some of the results of the report are published in the form of research papers in standard journals and the remaining results can be communicated for publication.

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